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CP Symmetry and Fermion Masses in $O(10)$ Grand Unification Models

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O(10) grand unification models which do not necessarily have an extra global symmetry are discussed, taking the model with one 10-plet in the Yukawa sector as an example. A strong correlation between mass ratios and CP is found. The mass relation $m_t/m_b = v_u/v_d$ is recovered when $G_W = 0$; and another special relation $m_t/m_b = G_E/G_W$ appears when $v_d = 0$, where $G_{E,W}$ are Yukawa coupling constants and $v_{u,d}$ are VEVs. To facilitate this discussion, a set of $O(10)$ γ- matrices is offered based on a physical representation of the spinors and that of the vector of the $SO(10)$ group. Flavor changing neutral currents in such models are also discussed.

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I. Motivations

The $SO(10)$ group is one of the most prominent groups for unification theories[1]. In most previous studies of $SO(10)$ grand unification theories, extra global symmetries are assumed either explicitly or implicitly. While these extra global symmetries can be justified from supersymmetry or superstring theories, a pure $SO(10)$ grand unification theory may still be of interest. For example, the flavor changing neutral currents (FCNC) in a $SO(10)$ model is somewhat subtle. Taking the model with one 10-plet mass provider (minimal O-ten models, MOTM) as an example, the 10-plet is decomposed into $SU(5)$ representations as $10 = 5_1 + 5_2^*$. The fermions 16 and 16* are decomposed as $16 = 10 + 5^* + 1$, and $16^* = 10^* + 5 + 1$. Therefore there are two sets of possible mass terms:

$$5_1 \ 10^* \ 5, \qquad 5_2^* \ 10^* \ 10^*;$$

and

$$5_1 \ 10 \ 10, \qquad 5_2^* \ 10 \ 5^*.$$

There will be no FCNC at low energies for MOTM with three generations of fermions, if only one of the two sets contributes. On the other hand, if only one of the 5-plets contribute, but both couple to fermions, as they must do because they are in the same multiplet, there can still be FCNC.

Indeed, one will see that the top-bottom mass ratio can be written as

$$|\frac{m_t}{m_b}| = \left\{ \frac{[Re(g_e v_1 + g_o v_6)]^2 + [Im(g_e v_6 + g_o v_1)]^2}{[Re(g_e v_1 - g_o v_6)]^2 + [Im(g_e v_6 - g_o v_1)]^2} \right\}^{\frac{1}{2}}. \tag{1}$$

Here, $v_1 = (v_u + v_d)/2$, $v_6 = (v_u - v_d)/2$ where v_u and v_d are the VEVs of 5_1 and 5_2^* respectively of the 10-plet Higgs. g_e and g_o are respectively coupling constants of $O(10)$ parity even and odd terms. The 10-d space reflection is not an element of the $SO(10)$ group. Therefore $O(10)$ is also of interest. One can see from this formula that in order to have $|m_t/m_b| \neq 1$, not only one needs both vacua v_1 and v_6 but also both couplings g_e and g_o. When there is a maximal CP mixing, $g_o = \pm i g_e$, the mass ratios will be adversely affected in the MOTM.

Two special cases are worth noting: 1) $v_1 = v_6$: This means that there is only one nonzero VEV, $v_u \neq 0$, $v_d = 0$, which is typical for a vector (the 10-plet) to develop VEV.

In this case, one obtains $m_t/m_b = (g_e + g_o)/(g_e - g_o)$. 2) The Yukawa couplings satisfy the condition for being self-dual, $g_o = g_e$. The definition of dual (denoted by E for convenience) and anti-dual (denoted by W) in $O(10)$ is similar to that of left-handed (L) and right-handed (R) in the Lorentz group. In this case one obtains a two Higgs doublet model with either supersymmetry or a global $U(1)$ symmetry[2] at low energies.

In general, the counterpart of a MOTM at low energies is a general two Higgs doublet model[3] with FCNC and a complicated relation between m_t/m_b and v_u/v_d. The second special case, particularly when it is resulted from supersymmetry, is widely applied for discussions of $SO(10)$ mass relations[4]. In this note we will analyze the $O(10)$ models without any constraints.

Therefore a one 10-plet Higgs $O(10)$ model (MOTM) may in general correspond to a two Higgs doublet model with FCNC. If more Higgs multiplets are involved in the Yukawa sector, then there will be more FCNCs. In either case, the Yukawa coupling constants can be complex, which may cause explicit CP violation. In addition to this, spontaneous CP violation due to a relative phase of the VEVs may appear in all the electro-weak interactions. In general, one may need eight U- matrices in order to diagonalize the mass matrices of the up-, down-, neutrino- and lepton- mass matrices: U_L^U, U_R^U; U_L^D, U_R^D; U_L^ν, U_R^ν; and U_L^l, U_R^l. All of them can be physically relevant; in other words, in addition to the CKM matrix $V_{CKM} = U_L^U U_L^{D\dagger}$; the matrix $V' = U_R^U U_R^{D\dagger}$ appears in the right-handed charged current gauge interactions; the matrix $\tilde{G}^U = U_L^U G_Y^U U_R^U$ leads to the scalar mediated FCNC interactions among up-type quarks, where G_Y^U is the matrix of Yukawa couplings of this scalar. Unless G_Y^U of a Higgs doublet is proportional to the corresponding mass matrix, FCNC mediated by this Higgs field is in general nontrivial. CP violation can in principle appear in any of above interactions. V' and $\tilde{G}^U$ and the alike represent physics beyond the CKM matrix[3, 5].

This work is devoted to the general relations among mass ratios, FCNC, and CP violation in an arbitrary $O(10)$ model. The MOTM will be taken as an explicit example. However, the results can be applied to any non-minimal $O(10)$ models. Before such a discussion, a set of explicit γ- matrices will be provided in Section II. The properties of mass operators will be discussed in detail in Section III. The special cases will be reviewed in terms of the

dual (E) and anti-dual (W) coupling constants. In Section IV, symmetries beyond $O(10)$ are discussed which may help to forbid the W term or the E term.

II. The $SO(10)$ Gamma Matrices and Mass Operators

There have been discussions on the group of $SO(10)$, since it was recognized as a potential candidate group for grand unification theories (GUT)[1, 6]. A different approach will be taken in this work. The components of the fundamental spinor and vector representations will be assigned first, in terms of the familiar quantum numbers, such as color, flavor, and $B - L$. Then the γ- matrices will be built up on this specific basis.

A fermion field can be seen as a sum of the left-handed and the right-handed parts $\psi = \psi_L + \psi_R, \quad \psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi$. A generic mass term is

$$\bar{\psi}_L \psi_R = \psi_L^{cT} C \psi_R = \psi_R^T C \psi_L^c. \tag{2}$$

where $\psi^c = C\bar{\psi}^T$, and $C = i\gamma_2\gamma_0$ is the C-matrix of the Lorentz group, $C^T = C^{-1} = -C$. From here on, all Lorentz group matrices will be underlined, in order to distinguish them from the $SO(10)$ matrices.

The present task is to find ten 32×32 matrices which satisfy the Clifford algebra:

$$\{\gamma_M, \gamma_N\} = \gamma_M \gamma_N + \gamma_N \gamma_M = 2\delta_{MN} I^{32}. \tag{3}$$

The anti-commuting relation (3) keeps its validity under orthogonal transformations $\gamma'_M = a_{MN}\gamma_N$. In addition one has a freedom to choose the components of the reducible spinor $2^5 = 16 + 16^*$.

The two irreducible spinors of $SO(10)$ are represented here by ψ and ψ^c. They are used to represent respectively Lorentzan left-handed and right-handed Weyl fields in one family of fermions

$$\psi^T = (u_L, d_L, d_R^c, u_R^c, \nu_L, e_L, e_R^c, \nu_R^c), \tag{4}$$

and the 16*- plet is just its charge conjugate. The color indices (from 1 to 3) for the quarks are suppressed. The arrangement of the components in (4) complies with a $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ decomposition of the $SO(10)$ spinor: $16 = (3, 2, 1, +1/3) + (3^*, 1, 2, -1/3) +$

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$(1, 2, 1, -1) + (1, 1, 2, +1)$, where the fourth number in each parenthesis is the $B - L$ quantum number[3]. The 32-d reduced representation is chosen as Ψ, $\Psi = \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}$.

It is convenient to first define 10 symmetric 16×16 matrices in two groups: α_i, β_p, $(i = 1, 2, 3, 4, 5; p = 6, 7, 8, 9, 10)$, where αs and βs mutually commute, while each groups make separate Clifford algebras,

$$\{\alpha_i, \alpha_j\} = 2\delta_{ij} I^{16}, \quad \{\beta_p, \beta_q\} = 2\delta_{pq} I^{16}; \quad [\alpha_i, \beta_p] = 0. \tag{5}$$

The gamma matrices are then simply

$$\begin{aligned} \gamma_I &= \begin{pmatrix} 0 & \alpha_I \\ \alpha_I & 0 \end{pmatrix}, & (I = 1, \dots 5) \\ \\ \gamma_P &= \begin{pmatrix} 0 & -i\beta_P \\ i\beta_P & 0 \end{pmatrix}. & (P = 6, \dots 10) \end{aligned} \tag{6}$$

As mentioned before, the Clifford algebra is also invariant under orthogonal transformations, which changes the components of a 10-plet, for example. Therefore, the specific form of gamma-matrices depends on how we choose the components of a 10-plet. Using the fermion symbols, we represent quantum numbers of the chosen basis components for 10-plet as the following [4]

$$\begin{aligned} H = \tfrac{1}{2} \\ (\nu_{1L}\nu_{2R}^c + \nu_{2L}^c\nu_{1R}, e_{1L}\nu_{2R}^c + e_{2L}^c\nu_{1R}, d_{1R}^{1c}\nu_{2R}^c + d_{2R}^1\nu_{1R}, d_{1R}^{2c}\nu_{2R}^c + d_{2R}^2\nu_{1R}, d_{1R}^{3c}\nu_{2R}^c + d_{2R}^3\nu_{1R}; \\ \\ i\nu_{1L}\nu_{2R}^c - i\nu_{2L}^c\nu_{1R}, ie_{1L}\nu_{2R}^c - ie_{2L}^c\nu_{1R}, id_{1R}^{1c}\nu_{2R}^c - id_{2R}^1\nu_{1R}, id_{1R}^{2c}\nu_{2R}^c - id_{2R}^2\nu_{1R}, id_{1R}^{3c}\nu_{2R}^c - id_{2R}^3\nu_{1R}), \end{aligned} \tag{7}$$

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[3]The $SU(5) \times U(1)$ decomposition can be reached by rearranging the components to the following form: $\psi^T = (u_L, d_L, u_R^c, e_R^c, d_R^c, \nu_L, e_L, \nu_R^c)$.

[4]The fermion symbols used here are for their quantum numbers. For example, the quantum numbers of a term $\nu_{1L}\nu_{2R}^c$ are: electric charge $Q = 0$, lepton charge $L = 0$, and $(T_{3L}, T_{3R}) = (\tfrac{1}{2}, -\tfrac{1}{2})$. The attached subscripts (1 ro 2) are useful to avoid the 10-plet to be self-conjugated. Readers who do not prefer this basis for 10-plet, which mixes components with opposite quantum numbers, may read the next section for other representations in which no Clifford algebra can be found though.

the α- and β- matrices on the basis (7) are

$$\alpha_1 = \left(\begin{array}{cc|cc|c}
 & I^3 & & & \\
I^3 & & & & \\
\hline
 & & & I^3 & \\
 & & I^3 & & \\
\hline
 & & & & \tau_1 \\
 & & & & \tau_1
\end{array}\right), \qquad
\beta_6 = \left(\begin{array}{cc|cc|c}
 & -I^3 & & & \\
I^3 & & & & \\
\hline
 & & & I^3 & \\
 & & -I^3 & & \\
\hline
 & & & & \tau_2' \\
 & & & & -\tau_2'
\end{array}\right);$$

$$\alpha_2 = \left(\begin{array}{cc|cc|c}
 & I^3 & & & \\
-I^3 & & & & \\
\hline
 & & & I^3 & \\
 & & -I^3 & & \\
\hline
 & & & & \tau_3 \\
 & & & & \tau_3
\end{array}\right), \qquad
\beta_7 = \left(\begin{array}{cc|cc|c}
 & -I^3 & & & \\
-I^3 & & & & \\
\hline
 & & & -I^3 & \\
 & & -I^3 & & \\
\hline
 & & & & -I^2 \\
 & & & & -I^2
\end{array}\right);$$

$$\alpha_3 = \left(\begin{array}{cc|cc|c}
 & \phi^3 & & & B_1 \\
-\phi^3 & & & & C_1 \\
\hline
 & & & \phi^3 & B_1 \\
 & & -\phi^3 & & C_1 \\
\hline
B_1^T & C_1^T & & & \\
 & & B_1^T & C_1^T &
\end{array}\right), \qquad
\beta_8 = \left(\begin{array}{cc|cc|c}
 & -\phi^3 & & & B_1 \\
\phi^3 & & & & C_1 \\
\hline
 & & & \phi^3 & -B_1 \\
 & & -\phi^3 & & -C_1 \\
\hline
B_1^T & C_1^T & & & \\
 & & -B_1^T & -C_1^T &
\end{array}\right); \quad (8)$$

$$\alpha_4 = \left(\begin{array}{cc|cc|c}
 & h^3 & & & B_2 \\
-h^3 & & & & C_2 \\
\hline
 & & & h^3 & B_2 \\
 & & -h^3 & & C_2 \\
\hline
B_2^T & C_2^T & & & \\
 & & B_2^T & C_2^T &
\end{array}\right), \qquad
\beta_9 = \left(\begin{array}{cc|cc|c}
 & -h^3 & & & B_2 \\
h^3 & & & & C_2 \\
\hline
 & & & h^3 & -B_2 \\
 & & -h^3 & & -C_2 \\
\hline
B_2^T & C_2^T & & & \\
 & & -B_2^T & -C_2^T &
\end{array}\right);$$

$$\alpha_5 = \left(\begin{array}{cc|cc|c}
 & \tilde\phi^3 & & & B_3 \\
-\tilde\phi^3 & & & & C_3 \\
\hline
 & & & \phi^3 & B_3 \\
 & & -\tilde\phi^3 & & C_3 \\
\hline
B_3^T & C_3^T & & & \\
 & & B_3^T & C_3^T &
\end{array}\right), \qquad
\beta_{10} = \left(\begin{array}{cc|cc|c}
 & -\tilde\phi^3 & & & B_3 \\
\tilde\phi^3 & & & & C_3 \\
\hline
 & & & \phi^3 & -B_3 \\
 & & -\tilde\phi^3 & & -C_3 \\
\hline
B_3^T & C_3^T & & & \\
 & & -B_3^T & -C_3^T &
\end{array}\right),$$

where

$$I^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \;
h^3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \;
\phi^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \;
\tilde\phi^3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (9)$$

and

$$I^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \;
\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \;
\tau_2' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \;
\tau_3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (10)$$

finally

$$B_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B_3 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix};$$

$$C_1 = \begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \end{pmatrix}, \quad C_3 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix}.$$

(11)

In all of these matrices, empty fields correspond to zeros.

There are some additional matrices which will be useful for the discussion of discrete symmetries. First, γ_{11} is defined as the product of all the ten gamma matrices,

$$\gamma_{11} = -i\gamma_1 \gamma_2 \cdots \gamma_{10} = \mathrm{diag}\left(I^{16}, -I^{16}\right). \tag{12}$$

Secondly the C-matrix is the product of the first five γ- matrices

$$C = \gamma_1 \gamma_2 \gamma_3 \gamma_4 \gamma_5 = \begin{pmatrix} 0 & I^{16} \\ I^{16} & 0 \end{pmatrix}. \tag{13}$$

γ_{11} can be used to construct derived γ- matrices $\tilde{\gamma}_N = \gamma_{11}\gamma_N$, which satisfy the same conditions for a Clifford algebra, except for a sign difference in normalization.

To find out tensor representations decomposed from the product of two 32-spinor representations Ψ_1 and Ψ_2, let us first define $\bar{\Psi} = \Psi^{cT} = \Psi^T C$. The a-th order tensor γ- matrices are

$$\Gamma^{(a)}_{N_1 \cdots N_a} = \frac{1}{a!}\gamma_{[N_1} \cdots \gamma_{N_a]}, \quad (a = 0, \cdots, 2n). \tag{14}$$

The bracket $[\cdots]$ for the subindices means anti-symmetrization. Obviously $\Gamma^{(10)} = i\gamma_{11}$. The $SO(10)$ anti-symmetric tensor representations are simply $\bar{\Psi}_1 \Gamma^{(a)} \Psi_2$, where a $\underline{C}$ of the Lorentz group is implied as is shown in Eq(2).

The second order γ- matrices $\Gamma^{(2)}$ are the $SO(10)$ infinitesimal group operators in the 32 reduced representation. They are all anti-Hermitian and block diagonalized, therefore one

has[5]

$$e^{\Gamma_{MN}^{(2)T} a_{MN}} C \Gamma^{(0)} e^{\Gamma_{PQ}^{(2)} a_{PQ}} = C. \tag{15}$$

5 among the 45 operators are diagonalized on the chosen basis. They are $D_I = i\Gamma_{I,I+5} = i\gamma_I \gamma_{I+5}$. Note that

$$
\begin{array}{rcl}
\sqrt{\frac{1}{2}}(D_2 + D_1) & = & \sqrt{8} T_{3R}, \\
\sqrt{\frac{1}{2}}(D_2 - D_1) & = & \sqrt{8} T_{3L}, \\
\sqrt{\frac{1}{3}}(D_3 + D_4 + D_5) & = & -\sqrt{3}(B - L), \\
\sqrt{\frac{1}{2}}(D_3 - D_4) & = & -\sqrt{8} I_3^{color}, \\
\sqrt{\frac{1}{6}}(D_3 + D_4 - 2D_5) & = & -\sqrt{8} Y^{color}.
\end{array}
\tag{16}
$$

Here, T_{3L}=diag[1/2, -1/2] for each left-handed doublet; I_3^{color}= diag[1/2, -1/2, 0] and Y^{color}= $\sqrt{\frac{1}{12}}$diag[1, 1, -2] for each color triplet.

The mass operators must be block off-diagonal, color-singlet, and electrically neutral. It is easily seen that

$$\gamma_1 \quad \text{and} \quad \gamma_6 \tag{17}$$

in 10 can contribute to fermion masses if a 10-plet Higgs H_N develops VEV in the first and the sixth positions[6]. There are four mass operators in 120. They are

$$\gamma_{1,6}(i)\gamma_2\gamma_7 = \gamma_{1,6}D_2; \quad \gamma_{1,6}(B - L). \tag{18}$$

All non-zero elements of these operator matrices in (17) and (18) have quantum numbers

$$(SU(3), T_{3L}, T_{3R}, B - L) = (1, \pm 1/2, \mp 1/2, 0). \tag{19}$$

There are four mass operators in 126 also. Two of them are

$$\gamma_{1,6}(B - L)D_2, \tag{20}$$

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[5]However, when one of the ten dimensions (say, the 10th) is time-like, part of Γ_{MN} are Hermitian due to the following definition: $\gamma_0 = i\gamma_{10}$ which is anti- Hermitian. One then has

$$e^{\Gamma_{MN}^{(2)T} a_{MN}} C \gamma_0 e^{\Gamma_{PQ}^{(2)} a_{PQ}} = C\gamma_0.$$

This is the main difference between $SO(9,1)$ and $SO(10)$ groups.

[6]The naturalness of developing VEVs at two specific positions of a 10-plet is a question subject to study[7].

which enjoy the same property as described in (19). The other two, from its quantum number analysis, are found all to have zero elements except those with quantum numbers $(1, 0, \pm 1, \mp 2)$, or $(1, \pm 1, 0, \mp 2)$ and the first one is normally used to give right-handed neutrinos huge Majorana masses in order that a "see-saw" mechanism may take place to render a tiny left-handed neutrino mass[8].

A linear combination of the operators in (17) and (18) or (20) can provide flexibility to produce desired quark-lepton Dirac mass relations, as applied in all previous works. Their properties are listed in Table 1, where subindices i, j represent generation (or family) numbers.

<center>**Table** 1 A complete set of $SO(10)$ mass operators</center>

operator	repr.	mass relation	symm in family ind.
γ_1	10	$M_{ij}^N = M_{ij}^U = M_{ij}^l = M_{ij}^D$	S
γ_6	10	$M_{ij}^N = M_{ij}^U = -M_{ij}^l = -M_{ij}^D$	S
$2\gamma_1(T_{3R} + T_{3L})$	120	$M_{ij}^N = M_{ij}^U = -M_{ij}^l = -M_{ij}^D$	A
$2\gamma_6(T_{3R} + T_{3L})$	120	$M_{ij}^N = M_{ij}^U = M_{ij}^l = M_{ij}^D$	A
$\frac{1}{\sqrt{3}}\gamma_1(B - L)$	120	$M_{ij}^N = -3M_{ij}^U = M_{ij}^l = -3M_{ij}^D$	A
$\frac{1}{\sqrt{3}}\gamma_6(B - L)$	120	$M_{ij}^N = -3M_{ij}^U = -M_{ij}^l = 3M_{ij}^D$	A
$\frac{2}{\sqrt{3}}\gamma_1(T_{3R} + T_{3L})(B - L)$	126	$M_{ij}^N = -3M_{ij}^U = -M_{ij}^l = 3M_{ij}^D$	S
$\frac{2}{\sqrt{3}}\gamma_6(T_{3R} + T_{3L})(B - L)$	126	$M_{ij}^N = -3M_{ij}^U = M_{ij}^l = -3M_{ij}^D$	S

The method used here to produce all the necessary matrices on a physical basis can be used for other SO groups.

III. Masses and CP Violation

The most general Yukawa term involving one 10 is

$$L^Y = \frac{1}{2} \left(g_e^{ij} \bar{\Psi}_i \gamma_N H_N \Psi_j + g_o^{ij} \bar{\Psi}_i \gamma_{11} \gamma_N H_N \Psi_j \right) + h.c. \qquad (21)$$

Note that the expressions $(\bar{\Psi}_i \gamma_N \Psi_j + \bar{\Psi}_j \gamma_N \Psi_i)$ and $(\bar{\Psi}_i i\gamma_{11} \gamma_N \Psi_j + \bar{\Psi}_j i\gamma_{11} \gamma_N \Psi_i)$ are real. Both terms in (21) can be CP even if $\mathrm{Im} g_e g_o^* = 0$.

According to Eq(21), all fermions may get masses and mix together. For the purpose of illustrating how up-down relation goes in $O(10)$ models, we write down explicitly the relevant

<center>9</center>

terms for the third family

$$
\begin{aligned}
L^Y \;=\; & Re(g_e H_1 + g_o H_6')\bar{u}_3 u_3 + Im(g_e H_6' + g_o H_1)\bar{u}_3 i\bar{\gamma}_5 u_3 \\
+\; & Re(g_e H_1 - g_o H_6')\bar{d}_3 d_3 - Im(g_e H_6' - g_o H_1)\bar{d}_3 i\bar{\gamma}_5 d_3 \qquad (22) \\
+\; & \text{charged current inter.s} + \text{leptonic counterpart,}
\end{aligned}
$$

with $H_6' = -iH_6$ and $g_{e,o} = g_{e,o}^{33}$. It is easy to check that when g_e and g_o are real, the Yukawa term is indeed CP even if ReH_1 and ReH_6' are assigned CP even while ImH_1 and ImH_6' are CP odd. The mass relation in (1) is obtained, with a substitution of $\langle H_1 \rangle = v_1, \quad \langle H_6' \rangle = \langle -iH_6 \rangle = v_6$.

To discuss the special cases, it is more convenient to use the dual representation. This is done by introducing the following two sets of reduced γ- matrices, which do not belong to any Clifford algebra,

$$
\gamma_N^E = \frac{1}{2}(1 + \gamma_{11})\gamma_N, \quad \gamma_N^W = \frac{1}{2}(1 - \gamma_{11})\gamma_N, \quad \gamma^{E\dagger} = \gamma^W. \qquad (23)
$$

The E and W project operators $(1 \pm \gamma_{11})/2$ are $SO(10)$ invariant. They separate 16 from 16^* in a 32 reduced representation. The Yukawa terms expressed in the E-W basis is

$$
\begin{aligned}
L^Y \;=\; & \tfrac{1}{2}\left(G_E^{ij}\bar{\Psi}_i\gamma_N^E H_N\Psi_j + G_W^{ij}\bar{\Psi}_i\gamma_N^W H_N\Psi_j\right) + h.c. \\
=\; & G_E^{33}\left(H_u\bar{u}_{3L}u_{3R} + H_d\bar{d}_{3L}d_{3R}\right) + G_W^{33}\left(H_d\bar{u}_{3L}u_{3R} + H_u\bar{d}_{3L}d_{3R}\right) + h.c. \qquad (24) \\
+\; & \text{leptonic and charged} + \cdots,
\end{aligned}
$$

where $H_{u,d} = H_1 \pm iH_6, \quad G_{E,W} = g_e \pm g_o$. H_u and H_d are respectively in 5 and 5^* of $SU(5)$ which are respectively up and down components of 10 (please compare with Eq (7)).

Let us now return to the special cases discussed in Section I.

Case 1), $v_d = \langle H_d \rangle = 0$: One obtains

$$
m_t/m_b = (g_e + g_o)/(g_e - g_o) = G_E/G_W. \qquad (25)
$$

It can give the phenomenological mass ratio with only one VEV. But if $Re g_e g_o^* = 0$ (which corresponds to a maximal CP phase when $g_e g*_o \neq 0$) one is forced to have the trivial $O(10)$ mass relation $|m_t| = |m_b|$.

The mass matrix for the up-type quarks and down-type quarks are, in the case of three families of quarks and leptons,

$$M^U = G_E^{ij} v, \quad M^D = G_W^{ij} v, \quad (i, \ j = 1, \ 2, \ 3) \tag{26}$$

while the Yukawa couplings for the H_d field, which does not develop VEV, are

$$Y^U = G_W^{ij}, \quad Y^D = G_E^{ij}. \tag{27}$$

Although all fermions obtain their masses from the one and only vacuum expectation value $v = v_u$, there does not necessarily exist a proportionality relation between the masses and the Yukawa coupling constants of the H_d field, unless $g_o = 0$ or $g_e = 0$. Therefore, the H_d mediated FCNC exist in general, as does new CP violation in the Yukawa sector, if $\text{Im} g_e g_o^* \neq 0$. When $g_o = 0$, half of the Higgs degrees of freedom decouple from the Yukawa sector.

Case 2), $G_W = 0$: H_u only provides up-type masses; and H_d only provides down-type masses. This case has attracted the most attention in the literature. But it raises the question of how one can forbid the G_W term by certain symmetries. The only candidate within the $O(10)$ group seems to be the $O(10)$ dual transformation $\gamma_N \to \gamma_{11} \gamma_N$. The E-type term is self-dual and the W-type term is anti-self-dual. An extra $U(1)$ global symmetry appears in the Yukawa sector, along with self-duality.

Another representation can separate $H_\pm$ at the beginning, where $H_{I\pm} = H_I \pm i H_{I+5}$. ($I = 1$ to 5) H_{I+} and H_{I-} are in 5_1 and 5_2^* respectively of $SU(5)$ and $10 = (5_1, 5_2^*)$ in this representation. The associated reduced γ- matrices are $\gamma_{I\pm} = (\gamma_I \pm i \gamma_{I+5})/2$. ($I = 1$ to 5) The P even $(G_E = G_W)$ Yukawa interaction is then

$$\bar{\Psi} \sum_{I=1}^{5} (\gamma_{I-} H_{I+} + \gamma_{I+} H_{I-}) \Psi. \tag{28}$$

This representation is convenient for discussions of charged currents and gauge interactions.

IV. Discussions

It has been explicitly shown that a general $O(10)$ grand unification model with one 10-plet Higgs provides a natural motivation for the most general two Higgs doublet model (2HDM)

at low energies as discussed in detail in Ref. 3. In a general MOTM, there must be FCNC, if trivial up-down $O(10)$ mass relations is to be avoided. It is easy to see that this behavior also appears when one Higgs 120 or 126 contributes to Dirac fermion masses, except that 120 contributes only inter-family masses.

In addition to self-duality, one can also rule out the W- current- H coupling by the use of: a) supersymmetry; b) an extra $U(1)$ quantum number; c) a discrete symmetry of an order higher than five; d) a complex nonabelian group.

Actually, certain amount of FCNC is tolerable within the accuracy of the present experimental data, as discussed in Ref. 3. While self-duality can make $m_b \neq m_t$, the most general condition for $m_b \neq m_t$ is

$$
\begin{array}{ccccccc}
& \text{Re}(g_e g_o^*) & \neq & 0, & & \text{Re}(v_1 v_6^*) & \neq & 0, \\
\text{or} & |G_W| & \neq & |G_E|, & & |v_u| & \neq & |v_d|.
\end{array}
$$

When FCNC is allowed there are possibilities to realize the desired up-down mass ratio by a combination of two VEVs and two coupling constants.

It is very interesting that within the realm of the explicit CP invariant MOTM, one can adjust the up-down mass ratio in one family by adjusting g_o/g_e and v_u/v_d. Therefore it is possible to find an $O(10)$ model with a CP invariant Lagrangian. In such a model, CP violation all will be spontaneous.

In conclusion, there is a correlation between explicit CP violation and fermion mass relation in an $O(10)$ grand unification model. The $O(10)$ Higgs multiplets which may contribute to fermion masses have two or more doublets whose CP transformation properties are different. The introduction of self-duality, or symmetries beyond the $O(10)$ group, is crucial for getting rid of flavor changing neutral currents if they are undesired. For convenience of this study, a new form of explicit $O(10)$ gamma matrices are given, based on physical representations of spinors and vectors.

Except for the $O(10)$ group, other $O(2n)$ ($n \geq 2$) and E_6 models may also have similar correlation between CP violation and masses.

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